Some Perspectives in RHIC Physics

RHIC Users Meeting, BNL, June 4, 2009

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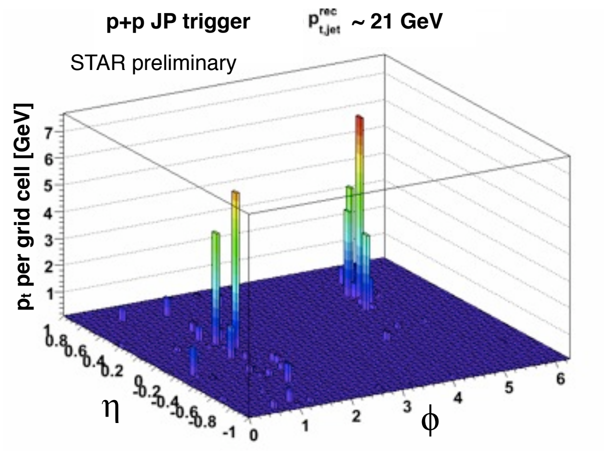
- **Not attempt at an exhaustive review; I'll offer observations in some arenas where pQCD meets RHIC:**

I.)



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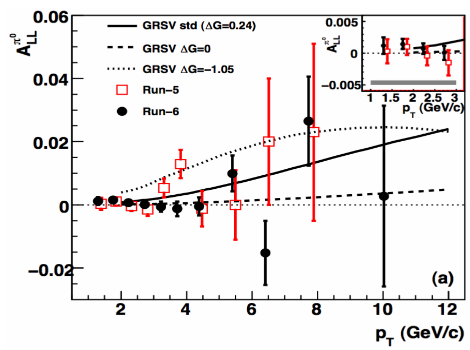
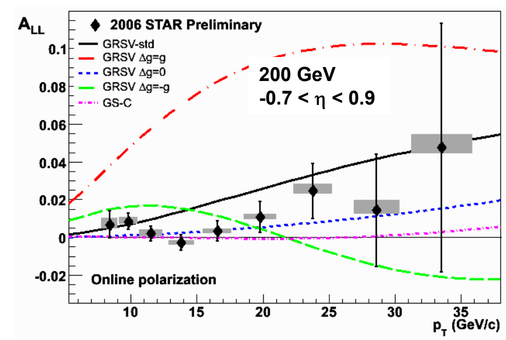
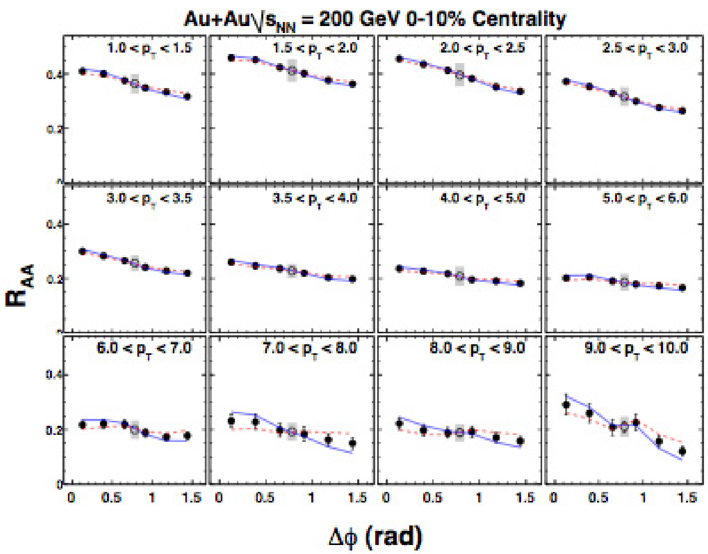
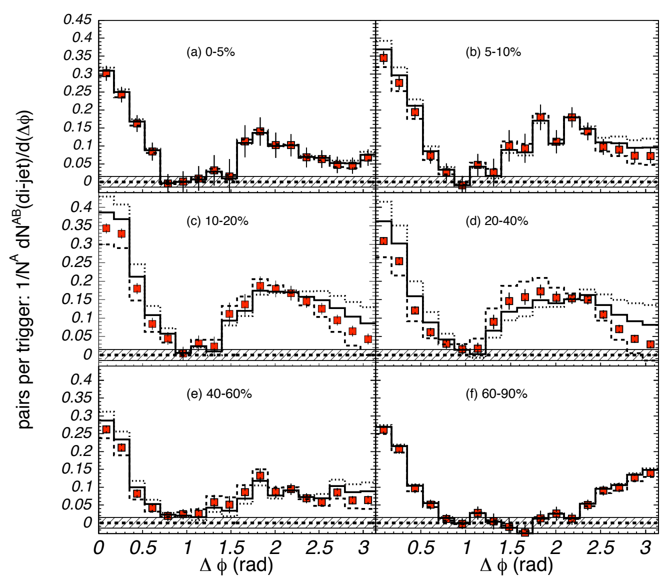




III.)



IV.)



I. Basics of "vacuum" pQCD

II. Factorization (when?) and polarized scattering

III. Jet structure and medium-dependence

IV. Inter-jet radiation and energy flow

I. Basics of "vacuum" pQCD

How we use asymptotic freedom

– Infrared safety & asymptotic freedom:

$$Q^2\ \hat{\sigma}_{\rm SD}(Q^2,\mu^2,\alpha_s(\mu)) = \sum_n c_n(Q^2/\mu^2)\ \alpha_s{}^n(\mu) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$
$$= \sum_n c_n(1)\ \alpha_s{}^n(Q) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

– $\mathrm{e}^+\mathrm{e}^-$ **total; jets**: a sum over collinear rearrangements and soft emission organizes all long-time transitions, which must sum to ≤ 1 by unitarity.

– What we're *really* looking at here (with local source J)

$$\sigma[f] = \lim_{R\to\infty} \int d^4x e^{-iq\cdot y} \int d\hat{n}\, f(\hat{n})$$

$$\times \langle 0|\, J(0) T[\hat{n}_i T_{0i}(x_0, R\hat{n}) J(y)]\, |0\rangle$$

(Sveshnikov & Tkachov 95, Korchemsky, Oderda & GS 96, Bauer, Fleming, Lee & GS 08, Hofman & Maldacena 08)

With T_{0i} the energy momentum tensor

– "Weight" $f(\hat{n})$ introduces no new dimensional scale

Short-distance dominated if all $d^k f / d\hat{n}^k$ bounded

– We have to ask only very "smooth" questions!

– Generalization: factorization

$$Q^2\sigma_{\mathrm{phys}}(Q,m,f) = \omega_{\mathrm{SD}}(Q/\mu,\alpha_s(\mu),f) \otimes f_{\mathrm{LD}}(\mu,m) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

μ = factorization scale;
m= IR scale **(m *may* be perturbative)**

– **"New physics" in ω_{SD}; f_{LD} "universal"**

– **Almost all collider applications. Enables us to compute the Energy-transfer-dependence in** $|\langle Q,\ \mathrm{out}|A+B,\ \mathrm{in}\rangle|^2$.

– **But again, requires a smooth weight for final states!**

Resummation?

– **Whenever there is factorization, there is evolution**

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q, m)$$

$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

– **Wherever there is evolution there is resummation,**

$$\sigma_{\rm phys}(Q, m) = \sigma_{\rm phys}(q, m) \ \exp\left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

– **For example:** $\sigma_{\rm phys} = \tilde{F}_2(Q^2, N)$, **DIS moment.**

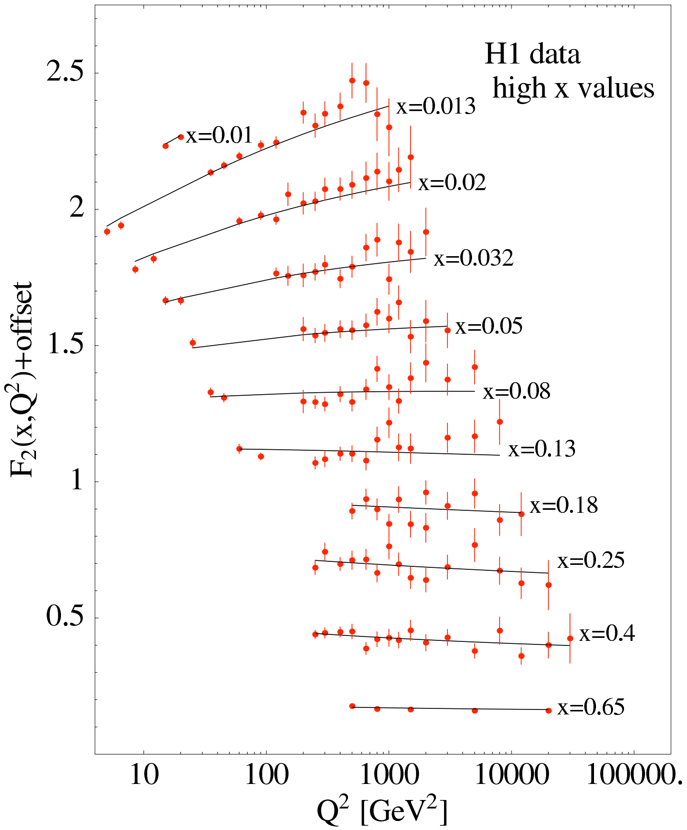
– & then we know $\tilde{P}(N,\alpha_s) = \gamma_N = \gamma_N^{(1)}(\alpha_s/\pi) + \ldots$, and we get

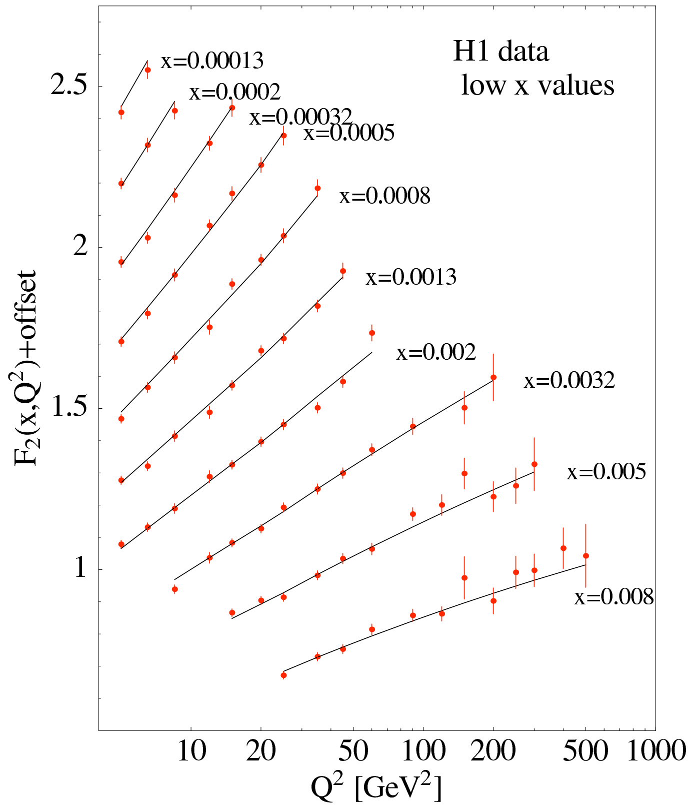
$$\tilde{F}_2(N,\mu) = \tilde{F}_2(N,\mu_0)\ \exp\left[-\frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2}\, \gamma(N,\alpha_s(\mu')) \right]$$

– and with $\alpha_s(\mu) = 4\pi/b_0 \ln(\mu^2/\Lambda^2_{\mathrm{QCD}})$, this is

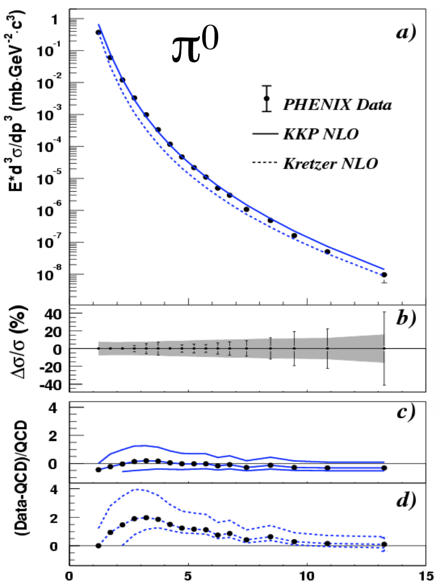
$$\tilde{F}_2(N,Q)\ = \tilde{F}_2 q/H(N,Q_0) \left(\frac{\ln(Q^2/\Lambda^2_{\mathrm{QCD}})}{\ln(Q_0^2/\Lambda^2_{\mathrm{QCD}})} \right)^{-2\gamma_N^{(1)}/b_0}$$

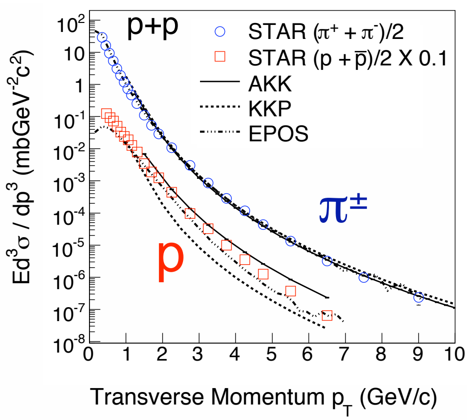
– **It works pretty well.** *Approximate scaling at moderate $\boldsymbol{x}$, pronounced evolution for smaller $\boldsymbol{x}$:*

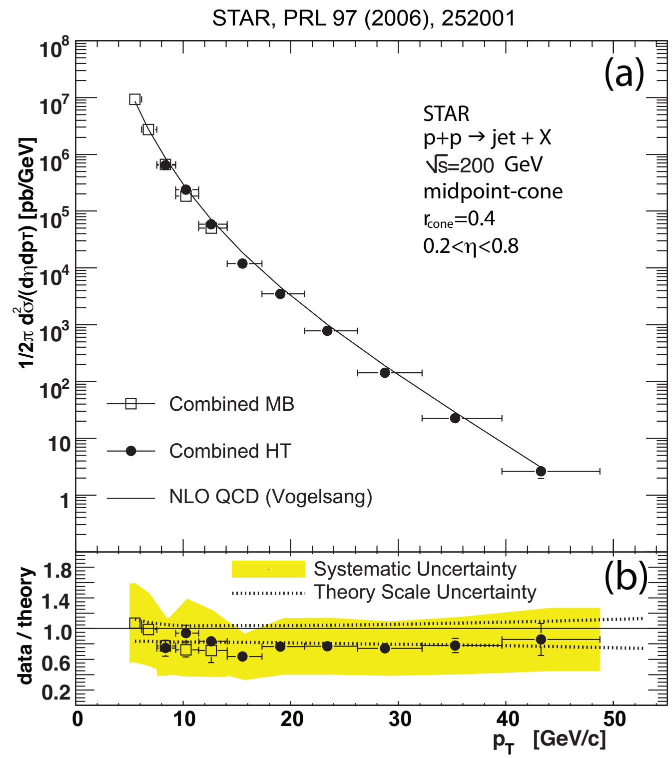




With these methods can describe both particles and jets in pp at 200 GeV . . .



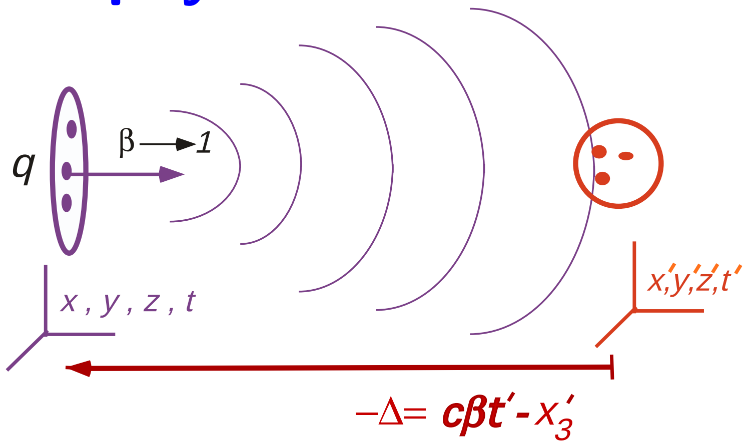




Especially for the single-particle inclusive cross sections, the range of agreement was a surprise. A great impetus for polarization and **HI studies. In ratios, at least we understand the denominator!**

II. Factorization (when?) and polarized scattering

- The physical basis in hadron collisions



$$\Delta \equiv x'_3 - \beta c t'$$

- Why a classical picture isn't far-fetched . . .

  The correspondence principle is the key to
  to IR divergences.

- An accelerated charge must produce classical radiation,
- and an infinite numbers of soft gluons are required
  to make a classical field.

Transformation of a <u>scalar</u> field:

$$\phi(x) \;=\; \frac{q}{(x_T^2 + x_3^2)^{1/2}} \;=\; \phi'(x') \;=\; \frac{q}{(x_T^2 + \gamma^2 \Delta^2)^{1/2}}$$

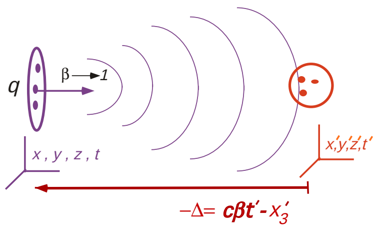
From the Lorentz transformation:

$$x_3 = \gamma(\beta c t' - x_3') \equiv -\gamma \Delta.$$

Closest approach is at $\Delta = 0$, i.e. $t' = \frac{1}{\beta c} x_3'$.

The scalar field transforms "like a ruler": At any fixed $\Delta \neq 0$, the field decreases like $1/\gamma = \sqrt{1 - \beta^2}$.

Why? Because when the source sees a distance x_3, the observer sees a much larger distance.



field	x frame	x' frame
scalar	$\frac{q}{\lvert\vec{x}\rvert}$	$\frac{q}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
gauge (0)	$A^0(x) = \frac{q}{\lvert\vec{x}\rvert}$	$A'^0(x') = \frac{-q\gamma}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
field strength	$E_3(x) = \frac{q}{\lvert\vec{x}\rvert^2}$	$E'_3(x') = \frac{-q\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
Gauge fields :	$E_3 \sim \gamma^0,$	$E_3 \sim \gamma^{-2}$

- **The "gluon" $\vec{A}$ is enhanced, yet is a total derivative:**

$$A^\mu = q\frac{\partial}{\partial x'_\mu}\ \ln\left(\Delta(t', x'_3)\right) + \mathcal{O}(1-\beta) \sim A^-$$

- **The "large" part of A^μ can be removed by a gauge transformation!**

- The "force" $\vec{\mathrm{E}}$ field of the incident particle does not overlap the "target" until the moment of the scattering.

- "Advanced" effects are corrections to the total derivative:

$$1-\beta \;\sim\; \frac{1}{2}\left[\sqrt{1-\beta^2}\right]^2 \;\sim\; \frac{m^2}{2E^2}$$

- Power-suppressed! These are corrections to factorization.

- At the same time, a gauge transformation also induces a phase on charged fields:

$$q(x) \Rightarrow q(x)\; e^{i\ln(\Delta)}$$

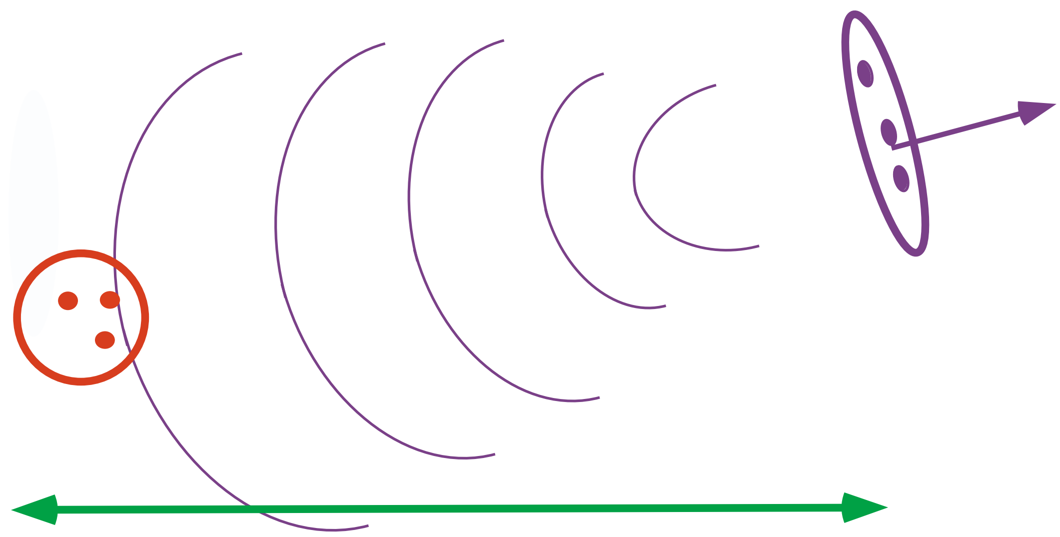
Cancelled if the fields are well-localized $\Leftrightarrow$ σ inclusive (smooth weight functions f).

- **Initial-state interactions decouple from hard scattering**
- **Summarized by multiplicative factors: the parton distributions**

  $\Rightarrow$ Cross section for inclusive hard scattering is IR safe, with power-suppressed corrections.

- **But what about cross sections where we observe specific particles in the final state? Single hadrons, dihadron correlations, etc?**

- **Much of the same reasoning holds:**



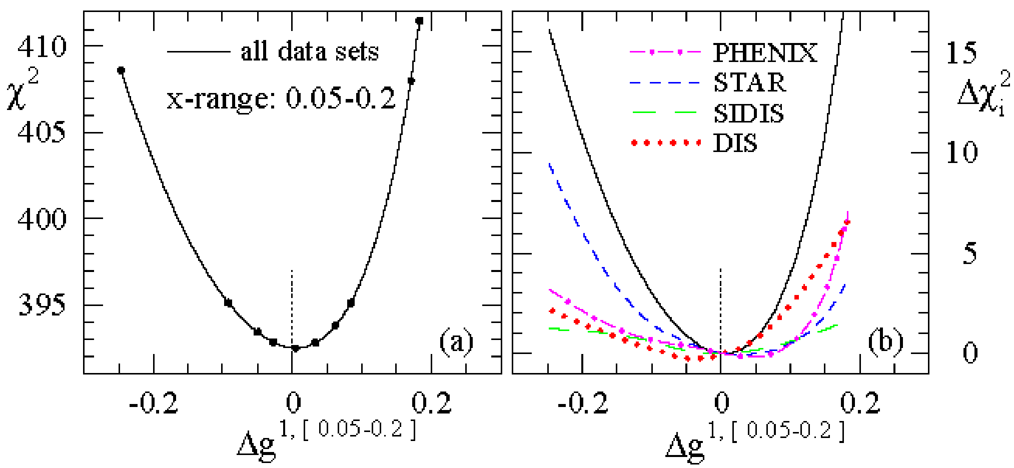
- **Subtle but important difference: Δ changes sign in the final state**
- **Then the gauge function in $\ln(\Delta)$ gets an imaginary part!**
- $q(x) \Rightarrow q(x)\, e^{i \ln(\Delta)}$ **no longer a pure phase!**
- **Mismatch between initial- and final-state interactions.**
- **Indicates physical effects in the final state.**

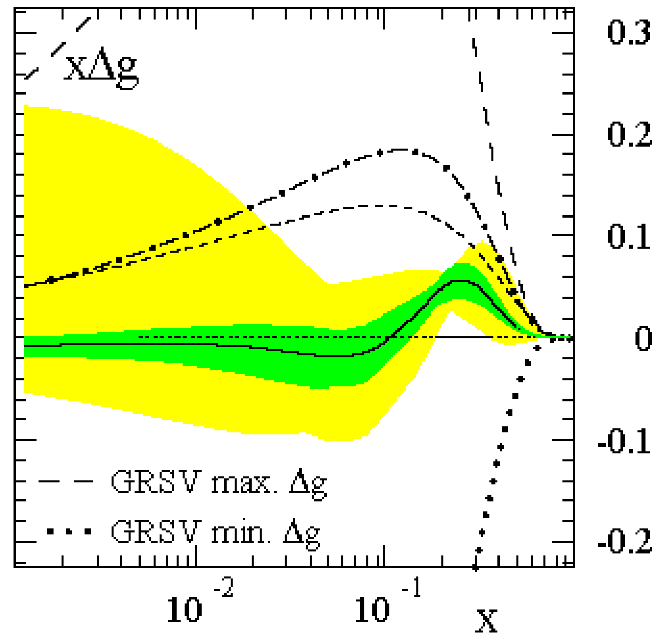
  (J. Collins & J.-W. Qiu)

- *Still* **cancels at high p_T for single hadrons, but not in general for distributions of momentum pairs.** (M. Aybat & GS)
- **But for single-particle inclusive . . .**
- **Interactions after the scattering are too late to affect large momentum transfer, creation of heavy particle, etc.**
- **Fragmentation of partons to jets too late to know details of the hard scattering: factorization of fragmentation functions.**

Lessons for polarized scattering

- **In general, universality between hadronic collisions and DIS/SIDIS holds only up to m^2/p_T^2 corrections.**
- **Fine for A_{LL} at leading power *and* for A_T (twist 3) at m/p_T DSSV et seq. . . .**





- **The spin does not *have* to be carried at the parton level**

- **For A_T in hadronic collisions, the use of Sivers and/or Collins functions found from DIS invite further theory developments; i.e. we don't really know what's going on yet.**
- **Twist-3 is the leading k_T/p_T contribution, with k_T a parton's transverse momentum.**
- **Sivers functions summarize all powers in k_T/p_T.**
- **Non-universal contributions are a window to non-factoring dynamics.**
- **Electron-ion collider will open new windows to all these questions: polarized and unpolarized, leading and higher twist/power.**

- **The gauge-theory analog of our classical argument is the universal soft-parton factor:**

  Real soft gluon k emitted by fast quark p, Dirac eq. gives:

$$\bar{u}(p)\,(-ig_s\,\gamma^\mu\,)\,\frac{\not{p}+\not{k}\;+\;m}{(p+k)^2\;-\;m^2}\;=\;\bar{u}(p)\,(-ig_s\,)\,\frac{p^\mu}{p\cdot k}$$
$$+\;(IR\;finite)$$

In a diagram p^μ will be contracted with a gluon propagator,

and in $p \cdot A = 0$ gauge, this term vanishes!

$$G^{\nu\mu}(k) = -\left(g^{\nu\mu}\;-\frac{p^\nu\,k^\mu\;+\;k^\nu\,p^\mu}{p\cdot k}\;+\;p^2\,\frac{k^\nu\,k^\nu}{(p\cdot k)^2}\right)$$

- **Notice this gauge depends on the momentum p.**

- **The origin of the "universality" of soft gluon interactions.**

- **But it is the same for every parton in a jet.**
- **But when $k^2 < 0$, this need not be the case, and we also have:**

$$\bar{u}(p)(-ig_s\gamma^\mu)\,\frac{\not{p}+\not{k}\ +\ m}{(p+k)^2\ -\ m^2}$$
$$= i\pi\delta(2p\cdot k + k^2)\bar{u}(p)\ (-ig_s)\ 2p^\mu$$

- **This is the QFT analog of the extra "phase" in the classical model.**

III. Jet structure and medium-dependence

- **One way**: First – find a jet. Then assign an axis $\hat{n}_J$: by minimizing $\sum_i E_i \cos\theta_{(i,\hat{n}_J)}$ for particles i in jet J.
- **Angularities** (C.F. Berger, Kúcs, GS, Magnea, Baur, Fleming, C.Lee. . . (2003 . . .))

$$\tau_a = \frac{1}{Q_J} \sum_{i\,\mathrm{in}\,N} p_{Ti}\, e^{-(1-a)|\eta_i|}$$

- p_{Ti}, η_i measured relative to thrust ($a = 0$) axis (can be chosen jet-by-jet).
- Broadening: $a = 1$; inclusive limit $a \to \infty$.
- For multijet final states, define η_i relative to closest jet.

- **Cross section is a convolution in contributions of each jet and a soft radiation function**

$$\sigma\,(\tau_a, Q, a) = H_{IJ} \int dt_s \prod_{\text{jets } i} \int dt_i\, S_{JI}(t_s) \prod_i J_i(t_i, p_{Ji})$$
$$\times\, \delta\Big(\sum_i t_i + t_s - \tau_a\Big)$$

- **Thus, general resummed cross section can be written as an inverse transform**

$$\sigma\,(\tau_a, Q, a) = \int_C d\nu\, \mathrm{e}^{\nu\, \tau_a}\, H_{IJ}\, S_{JI}(\nu) \prod_i J_i(\nu, p_{Ji})$$

in terms of $f(\nu) = \int_0^\infty dt\, e^{-\nu t}\, f(t)$ **.**
Llarge (Catani, Turnock, Trentadue, Webber (1990-92))

- **Three-way factorization $\Rightarrow$ CO/IR (Sudakov) resummation.** **Two logarithmic integrals exponentiate:**

$$J_i(\nu, p_{Ji}) = \int_0 d\tau_a \, \mathrm{e}^{-\nu\tau_{Ji}} \, J_i(\tau_{Ji}, p_{Ji}) = \mathrm{e}^{\frac{1}{2}E(\nu,Q,a)}$$

$$E(\nu, Q, a) = 2 \int_0^1 \frac{du}{u} \Big[\int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right) \left(\mathrm{e}^{-u^{1-a}\nu(p_T/Q)^a} - 1\right) + \frac{1}{2} B\left(\alpha_s(\sqrt{u}Q)\right) \left(\mathrm{e}^{-u(\nu/2)^{2/(2-a)}} - 1\right) \Big]$$

- **Expansion in $\alpha_s(Q)$ finite at all orders. The "cusp" function $A(\alpha_s)$ depends on color representation of the parent parton, only $B(\alpha_s)$ depends on its spin.**

Nonperturbative effects

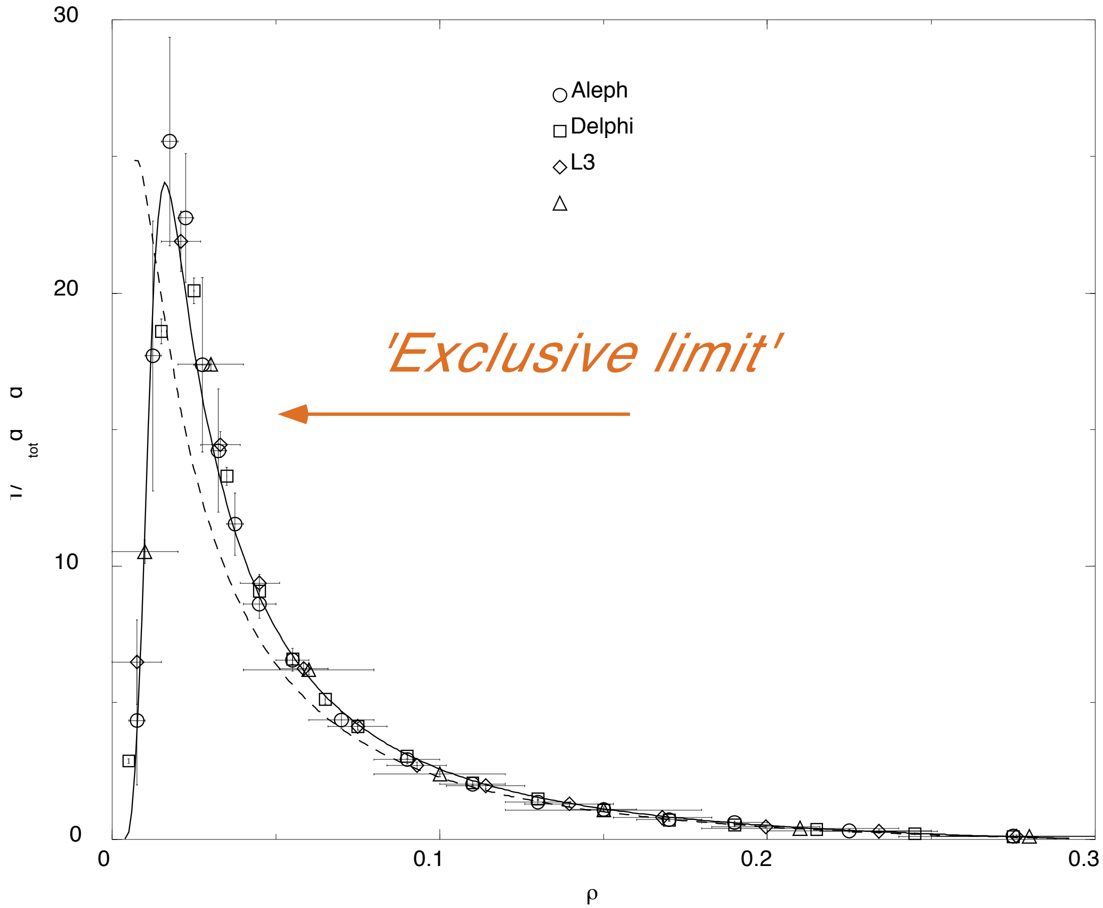
- **How to interpret expressions like ($a = 0$)**

$$E(\nu, Q, 0) = 2 \int_0^1 \frac{du}{u} \Big[\int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right) \left(\mathrm{e}^{-u\nu(p_T/Q)} - 1\right) + \frac{1}{2} B\left(\alpha_s(\sqrt{u}Q)\right) \left(\mathrm{e}^{-u(\nu/2)} - 1\right) \Big] \; ?$$

- Enter: nonperturbative scales in resummmed PT

  Phenomenologically important event at high energy.

- **Example: Heavy jet distribution at the LEP Z pole ($\sim \tau_0$).**
  (Korchemsky and Tafat (2000))



- **Dashed line: NLL resummed; solid line: nonperturbative "shape function" fit. What's that?**

- “Split” integral over p_T ($\alpha_s(p_T)$) in $E(\nu, Q, a)$:
  - $- p_T > \kappa \equiv E_{\rm PT}$,
  - $- p_T < \kappa$: expand in powers of $1/Q$

$$
\begin{aligned}
E(\nu, Q, a) &= E_{\rm PT}(\nu, Q, \kappa, a) \\
&\quad + \frac{2}{1-a} \sum_{n=1}^{\infty} \frac{1}{n\, n!} \left(-\frac{\nu}{Q}\right)^n \int_0^{\kappa^2} \frac{dp_T^2}{p_T^2}\, p_T^n\, A\left(\alpha_s(p_T)\right) + \ldots \\
&\equiv E_{\rm PT}(\nu, Q, \kappa, a) + \ln \tilde{f}_{a,{\rm NP}}\left(\frac{\nu}{Q}, \kappa\right)
\end{aligned}
$$

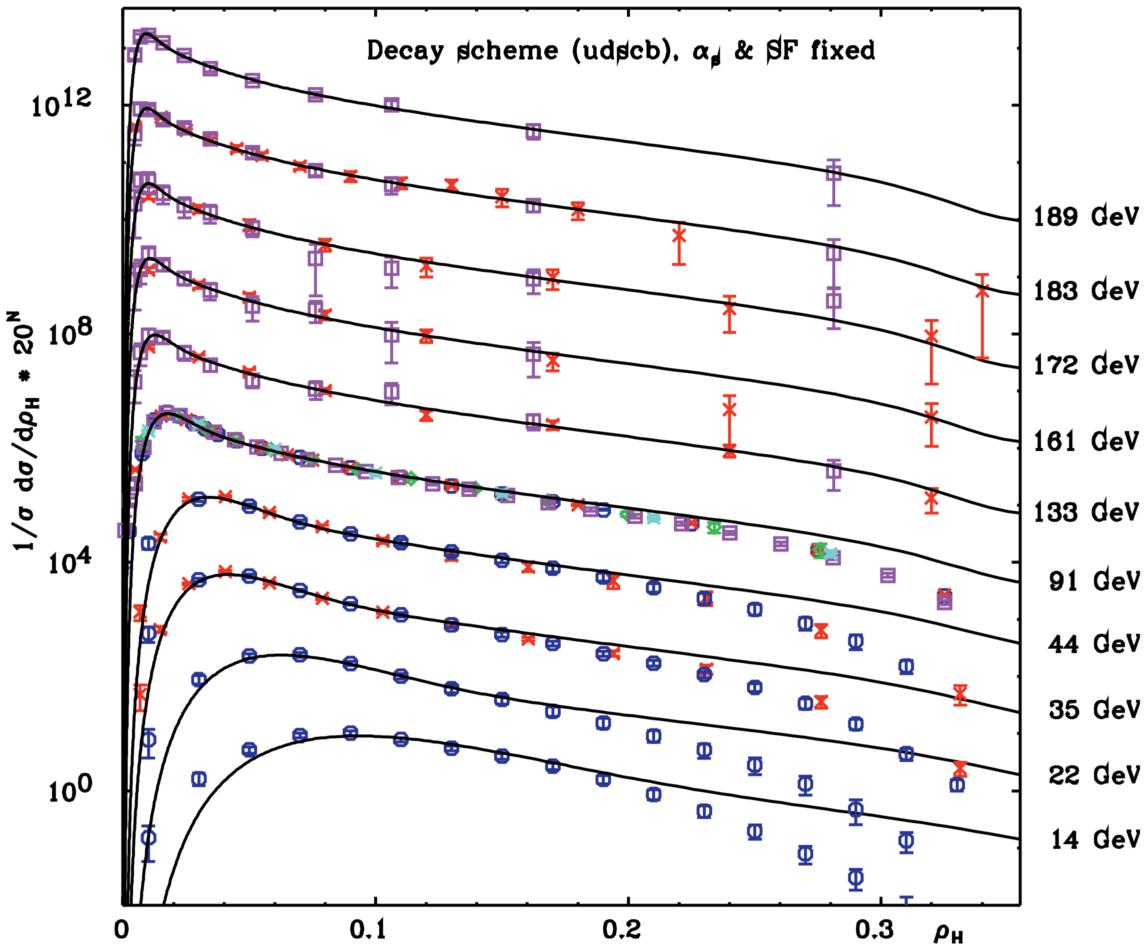
- Factors $\int_0^{\kappa^2} \frac{dp_T^2}{p_T^2}\, p_T^n\, A\left(\alpha_s(p_T)\right)$ normalize Q^{-n} power corrections
- Summarize in a factorized “shape function” $f_{a,NP}$ (additive in exponent E).

- **Shape function factorizes in moments $\longrightarrow$ convolution**

$$\sigma(\tau_a, Q) = \int d\xi f_{a,\mathrm{NP}}(\xi)\; \sigma_{\mathrm{PT}}(\tau_a - \xi, Q)$$

- $\mathrm{e^+e^-}$**: fit at $Q = M_{\mathrm{Z}}$ $\Rightarrow$predictions for all Q, any (quark) jet.**
- **Portable to jets in hadronic collisions**
- **and will be sensitive to gluon/quark origin of the jet.**
- **Might seem artificial – but the cusp function $A(\alpha_s)$ is universal and can even be studied at strong coupling in SYM . . . although its nonperturbative power corrections are purely "nonconformal", i.e. depend essentially on the running of the QCD coupling. Which is good, not bad.**

- **Shape function phenomenology for thrust at LEP.**



(Korchemsky,GS, Belitsky; Gardi Rathsman, Magnea, C.Lee . . . (1998 . . .))

L3 data on $a \neq 0$ will appear.

Can this be portable to HI jets?

- **Well, in**

$$E(\nu, Q, 0) \sim 2 \int_0^1 \frac{du}{u} [\int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right) \left(e^{-u\nu(p_T/Q)} - 1\right)]$$

u is conjugate to $1/tQ$, with t the "formation time" for gluon emission. So in a sense, E "tells a series of stories", of all possible emissions that take time t:

$$E(\nu, Q, 0) = 2 \int_0^\infty \frac{dt}{t} [\int_{Q/t}^{1/t^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right) \left(e^{-u\nu(p_T/Q)} - 1\right) + \frac{1}{2} B\left(\alpha_s(\sqrt{u}Q)\right) \left(e^{-u(\nu/2)} - 1\right)]$$

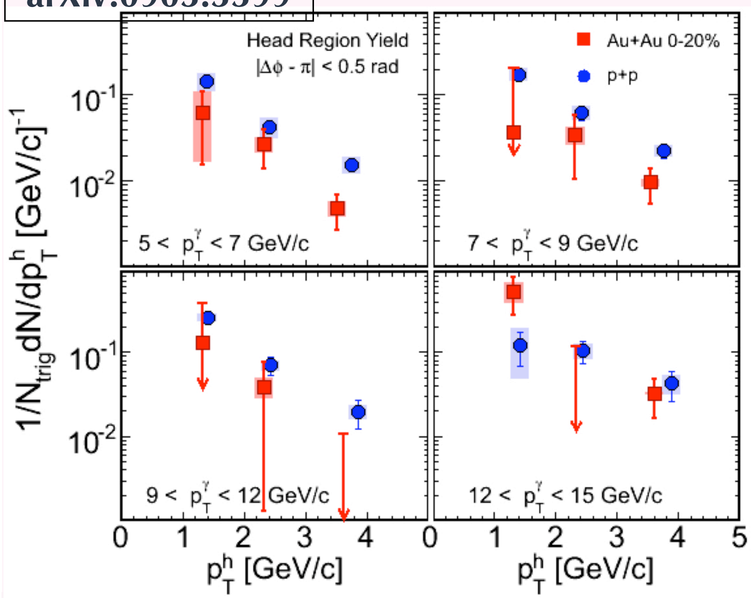
- **All these stories (like the power corrections) are additive in $E(\nu, Q, a)$.**

- **In principle, an analysis of shapes in pp and AA for angularities or other cleverly-chosen event shapes could provide the transition between the vacuum cusp function A and the quantum history of fast partons in the strongly interacting medium.**

- **This information is surely imprinted in jet correlations on the near-side and far-side and in photon-jet studies.**

Away side suppression

arXiv:0903.3399



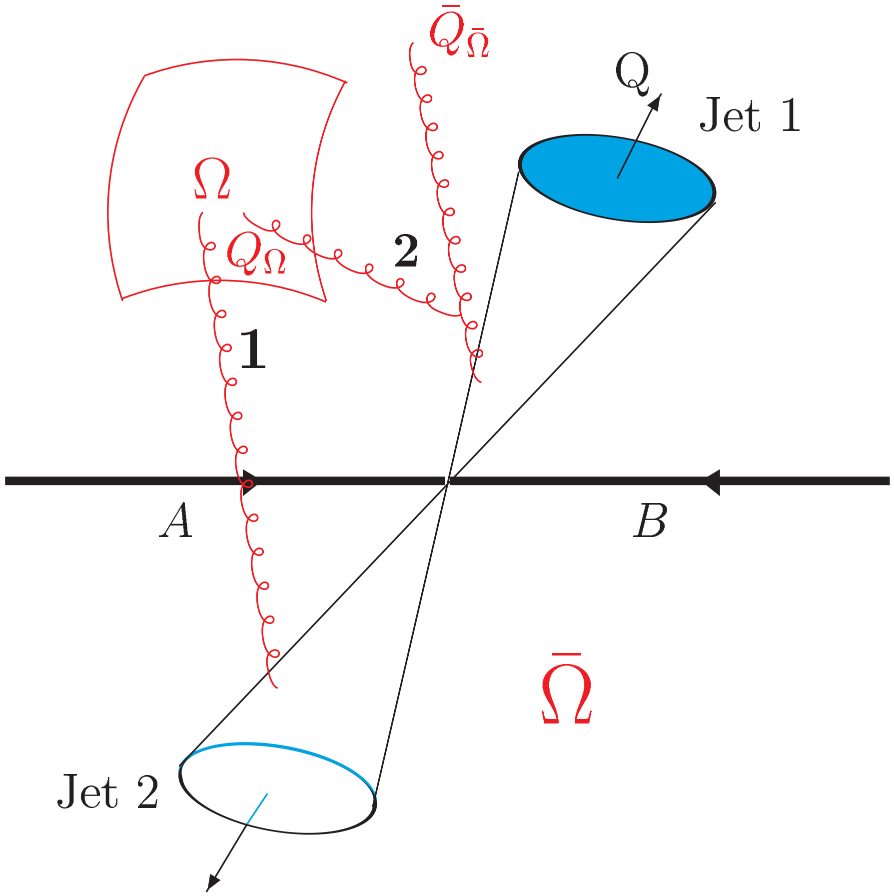
$$z_T = \frac{p_T^{hadron}}{p_T^{trigger}}$$

Fragmentation function:

$$D_q(z_T) = \frac{1}{N_{evt}} \frac{dN(z_T)}{dz_T}$$

$$I_{AA} \approx D_{AA}(z_T) / D_{pp}(z_T)$$

IV. Inter-jet Radiation: color and energy flow



– **Trigger on 2 or more jets. Measure distribution $\Sigma_\Omega(E)$**

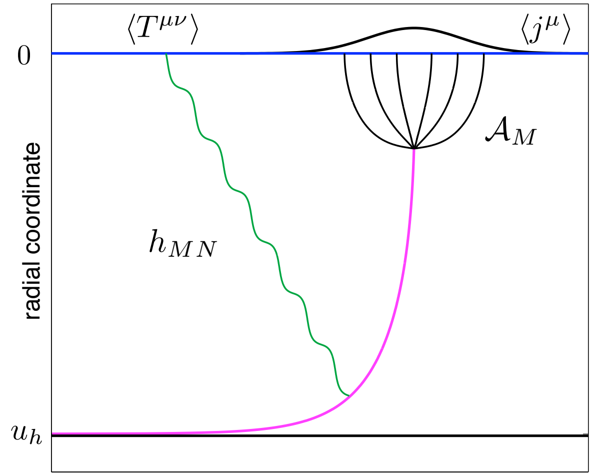
– **An example of multiple energy correlations:**

$$\Sigma[f] \sim \lim_{R\to\infty} \int d^4x e^{-iq\cdot y} \prod_i \int d\hat{n}_i\, f_\Omega(\hat{n}_1 \ldots)$$

$$\times \langle I|\, J(0) T[\prod_i \hat{n}_i T_{0i}(x_0, R\hat{n}) J(y)]\, |I\rangle$$

for some function of directions $f_\Omega(\hat{n}_1 \ldots)$ that demands 2 jets, and measures energy in Ω.

– **This is studied in duality-based models of energy-loss at strong coupling!**



(Chesler, Jensen, Karch, Yaffe (2008))

– There are many choices for such a cross section, including

– a) Simplest: inclusive in $\bar{\Omega}$ $\rightarrow$ Total number of jets ≥ 2 but not otherwise fixed.

– b) Correlation with an event shape τ_a . . . : fixes number of jets $\rightarrow$ "simple" factorization

(C.F. Berger, Kúcs, GS (2003), Dokshitzer, Marchesini (2003))

– for a): Number of jets not fixed: *much* more complex.
Summarizes many factorizations $\Rightarrow$ nonlinear evolution

(Banfi, Marchesini, Smye (2002)) LL in E/Q, large-N_c (all $\Sigma = \Sigma(E)$)

$$\partial_\Delta \Sigma_{ab} = -\partial_\Delta R_{ab}\, \Sigma_{ab} + \int_{k\in\bar{\Omega}} dN_{ab\to k}\, (\Sigma_{ak}\Sigma_{kb} - \Sigma_{ab})$$

$$dN_{ab\to k} = \frac{d\Omega_k}{4\pi}\, \frac{\beta_a\cdot\beta_b}{\beta_k\cdot\beta_b\,\beta_k\cdot\beta_a}, \qquad R_{ab} = \int_E^Q \frac{dE'}{E'}\, \int_\Omega dN_{ab\to k}$$

– Origin of the nonlinearity
  * $\partial_\Delta = E\partial_E$
  * ∂_E requires a “hard” gluon k
  * New hard gluon acts as new, recoil-less source
  * Large-N limit: $\bar{q}(a)G(k)q(b) \longrightarrow \bar{q}(a)q(k) \oplus \bar{q}(k)q(a)$

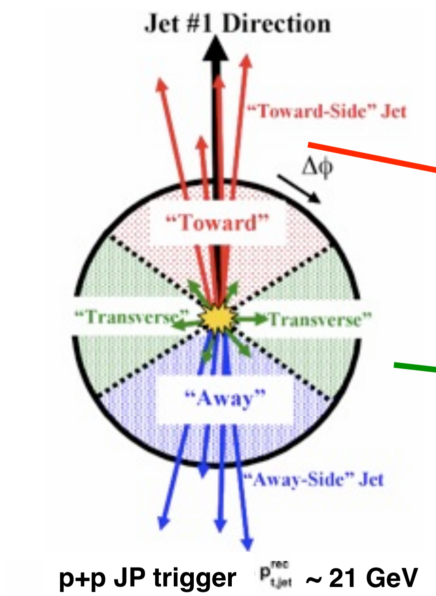
– VERY Intriguing relation to small-x saturation & BK Eqn.
(Weigert (2003), Hatta (2008,9) $\leftrightarrow$ strong coupling)

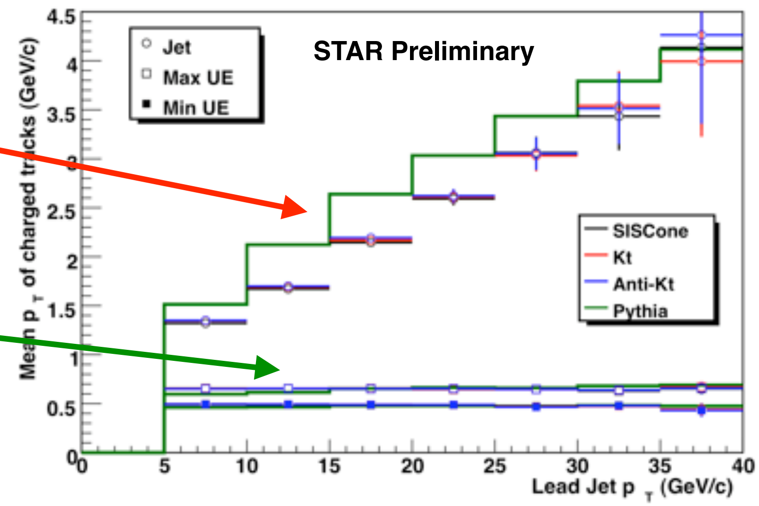
- For b) Correlation with event shape τ_a . . . :
  fixes number of jets
  - Keep $\tau_a Q \sim E_\Omega$ (BKS), Resum as above:

$$\frac{d\sigma}{dE_\Omega d\tau_a} \sim S(E_\Omega/\tau_a Q)\ \frac{d\sigma_{\rm resum}}{d\tau_a}$$

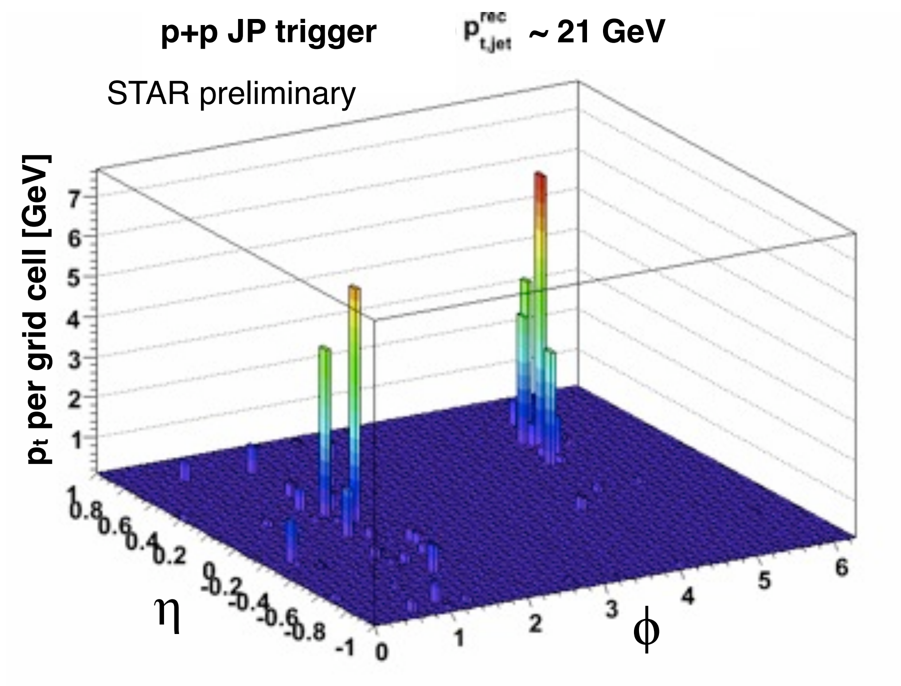
  - Limit $E_\Omega/\tau_a Q \longrightarrow 0$ (DM): use nonlinear evolution for S
  - Influence of color flow on energy flow at wide angles
    (Dokshitzer, Khoze, Troyan, Mueller . . .)
  - Applications to rapidity gaps
    (Oderda, GS; Appleby, Seymour, Sjodahl (2003 . . . 2009))

- **Interjet multiplicity studies at CDF: slow increase with jet energy**



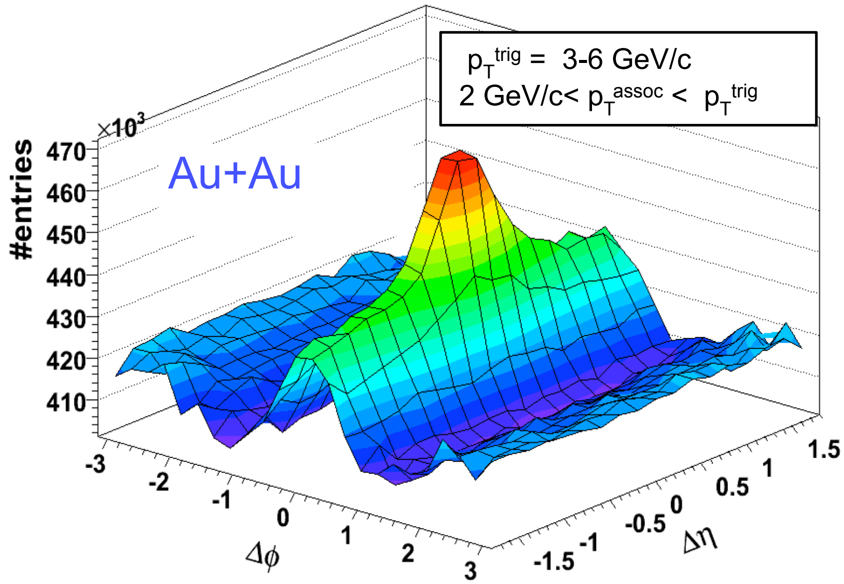


- **Striking absence of radiation into the ϕ-gap**

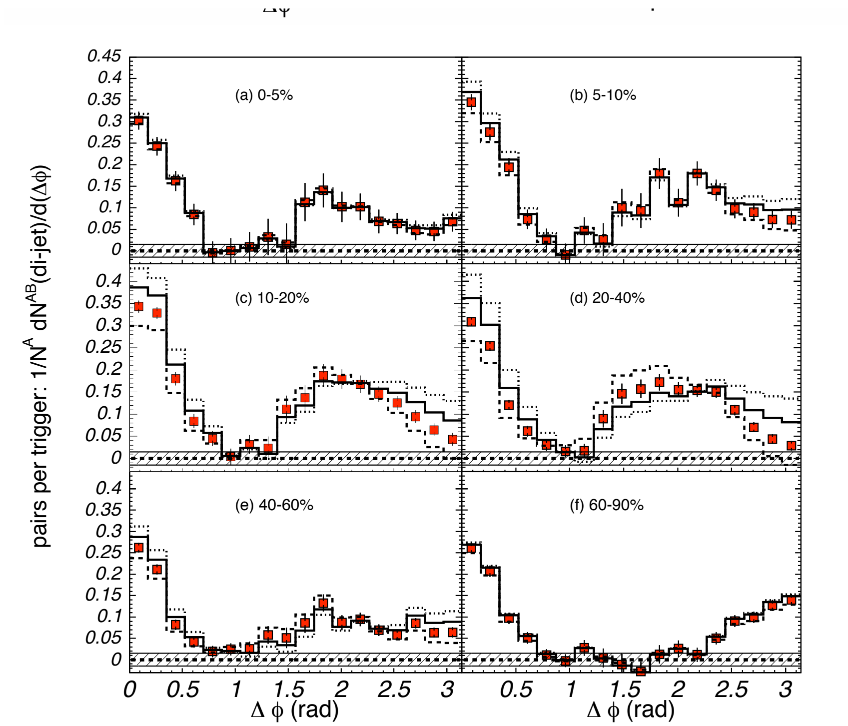


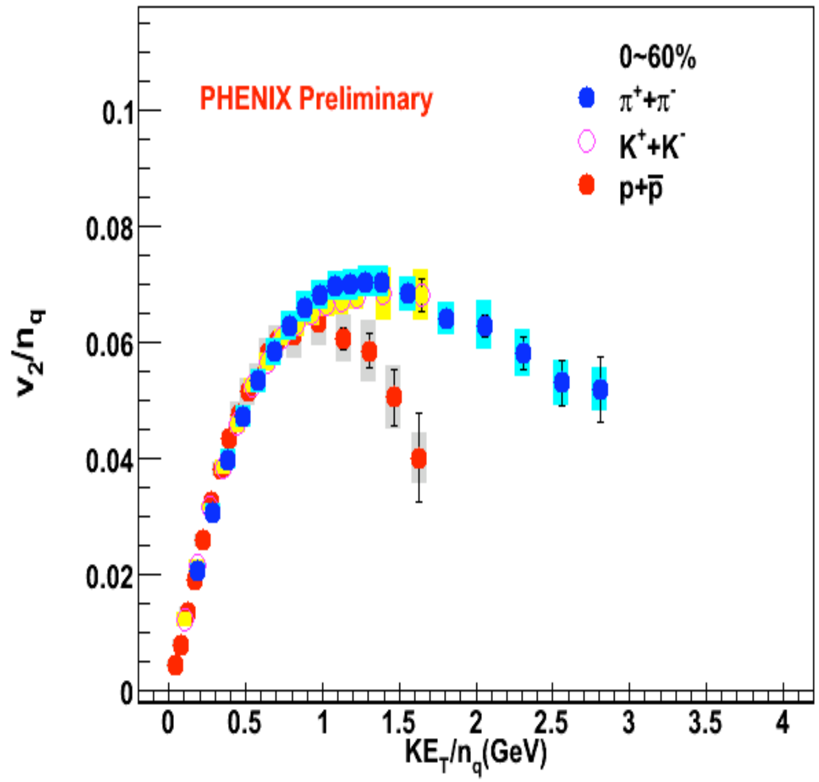
- Different from generic e^+e^- jet pair
- The probable difference: in e^+e^- the pair forms a color singlet dipole
- In hadronic collisions, the generic high-p_T pair of partons must find dipole matches in the forward direction.
- Anomalous dimension associated with hard/soft/collinear factorization encode this information but much remains to be done.

- **Earliest radiation may tend to be point towards the beam directions (larger $|\eta|$). An interesting coincidence with the "ridge" direction . . .**



- **Who knows? We may find a unified picture from radiation histories based on the cusp anomalous dimension all the way, to “jet splittings” and v_2:**





No Conclusion . . .

except congratulations on an astonishing range of discoveries, which opens too many doors for any single talk!